A method for road description based on map data and road measurements

White Paper

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www.vipsimulation.se
Preface

This White Paper has been written as a part of the Known Roads project. Known Roads was a collaborative project between Swedish National Road and Transport Research Institute (VTI), Dynagraph, Volvo Trucks, Volvo Car Corporation and Swedish Road Marking Association (SVMF) within the ViP Driving simulation centre (www.vipsimulation.se).

The scope of Known Roads was to develop virtual representations of real roads to be used in driving simulators. These simulator roads can be used in studies where similarity between a simulator road and the real road is of benefit.

The White Paper is a project deliverable intended for partners within the ViP consortium. It is complementary to the project report presenting the work and results of Known Roads (ViP publication 2015-2; Nåbo et al., 2015b).

The Known Roads project has been financed by the competence centre ViP, i.e. by the ViP partners and the Swedish Governmental Agency for Innovation Systems (Vinnova).

The software developed in the project is available at the ViP common software platform ViPForge, https://www.vipforge.se/. Access is granted to ViP partners by the ViPForge administrator Jonas Andersson Hultgren at VTI (jonas.andersson.hultgren@vti.se).

The White Paper work was undertaken by Mattias Hjort and Laban Källgren at VTI.

Gothenburg, November 2015

Arne Nåbo
Project manager
Quality review

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Abbreviations

GPS
Global Positioning System.

G1
Curves sharing common tangent at joint points.

G2
Curves sharing common centre of curvature at joint point.

C0
Splines without continuous derivative.

C1
Splines with continuous derivative.

NVDB
Nationell vägdata (National Road Database).

OpenCRG
Open source project including a tool-suite for the creation, modification and evaluation of road surfaces, and an open file format specification CRG (Curved Regular Grid).

OpenDRIVE
Open format specification to describe a road network's logic.

RST
Road Surface Tester.

ViP
Driving Simulation Centre. A competence centre administered by VTI.

VTI
Swedish National Road and Transport Research Institute.

XML
Extensible Markup Language.
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A method for road description based on map data and road measurements – White Paper

by Mattias Hjort (VTI) and Laban Källgren (VTI)

Executive summary

Simulation has become a useful tool for vehicle manufacturers in the design and assessment of new vehicles. Using an advanced driving simulator in early design phases may reveal defects in the construction, which otherwise would not be noticed until full scale prototyping, and thereby reduce development time and costs.

In most driving simulator studies the road design does not have to conform to an existing road. But for some purposes it may be crucial that the virtual road describes the real road as close as technically possible. Examples are vehicle vibration studies, fuel consumption evaluation with respect to driver behaviour, and comparisons between driving on the virtual and real road. The required level of detail of different road features varies between applications (projects). Parameters like road curvature, inclination, elevation and crossfall, the surrounding terrain as well as road imperfections and unevenness may be of more or less importance and therefore have to be modelled with high or lower accuracy.

In the Known Roads project, a real road, the “Gothenburg triangle” (Gothenburg-Borås-Alingsås-Gothenburg), was modelled as realistically as possible. Road curvature, inclination and crossfall as well as the surrounding terrain were considered important.

This White Paper is complementary to other deliverables from the Known Roads project. It presents in more detail the method of recreating existing roads in a simulation environment, developed by VTI in the project. The White Paper concerns specifically the difficulties of merging different data sources into a road representation, and describes the problems that were encountered and the solutions that were developed for implementing an existing road in the driving simulator.

Mathematical definitions and deductions of formulas are collected in the Appendix.
1. Introduction

This White Paper describes in more detail the method of recreating existing roads in a simulation environment, developed by VTI in the ViP project Known Roads\(^1\). The White Paper is complementary to the project report presenting the work and results of Known Roads (Nåbo et al., 2015b), but written so that it can be read and understood without having read that report. Thus, some information already presented in Nåbo et al. (2015) is repeated here.

An overview of the work in the Known Roads project is given in chapter 2, while the subsequent chapters in more detail describe the problems that were encountered and the solutions that were developed. An Appendix with mathematical definitions and deductions of formulas ends this White Paper. All mathematical formulas, except the general definitions and the description of Bezier curves, have been developed by VTI.

\(^1\) [http://www.vipsimulation.se/?page=mt10:-known-roads&pid=288](http://www.vipsimulation.se/?page=mt10:-known-roads&pid=288)
2. Overview

Virtual prototyping and assessment by simulation has become a useful tool for vehicle manufacturers to reduce costs and development times. Specifically, using an advanced driving simulator early in the design phase may reveal defects in the construction, which otherwise would not be noticed until full scale prototyping.

In most driving simulator studies, the road design does not have to conform to an existing road. For some purposes, however, it may be crucial that the virtual road describes the real road as close as technically possible. This could be vehicle vibration studies or fuel consumption evaluation with respect to driver behaviour, where road inclination and unevenness must be modelled with high accuracy. The required level of detail of the road varies between projects. As an example, the Electrified Road project (Näbo, et al., 2015a) needed an accurate description of the road between Borås and Gothenburg, with correct road curvature and elevation, as well as of the surrounding terrain, while road imperfections and short wave undulation were of less importance.

For the Known Roads project, it was considered important to model a real road as realistically as possible in order to facilitate comparisons with real driving on the road. This applies not only to simulation of the road surface conditions, but also road curvature, inclination and crossfall as well as the surrounding terrain. This White Paper concerns the difficulties of merging different data sources into a road representation.

2.1. Road simulation

To recreate a specific road several data sources were used. The main data was retrieved from measurements with the VTI Road Surface Tester\(^2\) (RST). The RST (see Figure 1) is a device that has been developed with the aim of measuring the road condition for maintenance purposes. It is equipped with many sensors. The road surface texture and short wave undulation are measured with 17 infrared laser sensors placed at the front of the vehicle, each measuring the road surface height with a resolution of 0.03 mm.

![VTI Road Surface Tester (RST).](image)

The RST vehicle is also equipped with accelerometers, inclinometers and a rate gyro, which give information of road curvature, crossfall and general elevation. A differential GPS (so called DGPS) relates the measurement data to specific road positions. Measurements can be carried out at a speed of 70 km/h with a sampling length of 0.1 meter for the measurement instruments. Shorter sampling length is possible at lower measurement speeds.

In the Known Roads project, the RST has been used for measuring several roads in the vicinity of Gothenburg, Sweden, with the purpose of road modelling. The roads, see Figure 2, were mostly two-lane rural roads, but in some cases also motorways and multilane city roads. Measurements with the RST were conducted in both directions at a speed of 70 km/h and with a sampling length of 0.1 m.

\(^2\) http://www.vti.se/sv/forskningsomraden/drift-och-underhall/matning-av-vagstandard/

ViP PM 2015-2
This will not give any information about the road surface texture, but the general unevenness of the road will be well captured. Wavelengths of 0.2 m can thus be measured, which when travelling at 20 m/s (72 km/h) can induce vibrations up to 100 Hz in the vehicle.

Figure 2. The roads measured with the RST vehicle were Road 40 Göteborg-Borås, Road 180 Borås-Alingsås, Road E20 Alingsås-Göteborg and the “Töllsjö” Road in-between these.

For an even better description of the road’s curvature and elevation it was decided to use also external information, (see Figure 3). The horizontal information about the road is described accurately in the Swedish National Road Database\(^3\) (NVDB), where GPS coordinates have been measured along the road at high precision to produce a two-dimensional description of the Swedish road network. The absolute road elevation was retrieved from flight measurements of the landscape along and surrounding the road, supplied by Lantmäteriet which is the organization in Sweden responsible for the national maps. Height data was given in a raster with 2x2 meter grids.

Figure 3. Input sources and output road model.

\(^3\) Nationella vägdatabase: http://www.nvdb.se/
As illustrated in Figure 3, three different sources of road information had to be merged into one road model describing the roads horizontal properties (curvature), elevation and surrounding terrain, and finally the road surface itself. The process that was developed is described below.

2.2. Horizontal properties

Although state of the art GPS equipment was used in the RST vehicle, it was clear from inspection of the measured GPS positions in Google Earth that the accuracy of the data (one measurement series for each lane) was not good enough for modelling of the road curvature. Thus, data was acquired from the Swedish National Road Database (NVDB), where GPS coordinates have been measured along the centreline of the roads at high precision.

Google can also be used in order to find a road’s alignment in the world, by making a travel direction request between the ends of the road, either in Google Maps or Google Earth. The direction can be exported to a kml-file (in xml format) in order to be parsed in any software. The coordinates are GPS points in the WGS84 system. The data set of Google is like NVDB not uniformly sampled, i.e. relative few data points constitute the road path accurately enough. However, we have seen examples where Google’s direction deviates from the true alignment (according to NVDB).

The horizontal properties of a road can be described by its curvature, which is the inverse of the curve radius. The curvature is mathematically defined in the Appendix. In theory, a road for simulations can be constructed using any curve shape. This may, however, lead to roads that are difficult to drive on, and it makes sense to use curve shapes that are used in road construction.

A road is normally projected and constructed using three kinds of elements: straight sections (zero curvature), circle segments (constant curvature) and clothoids (curvature changes linearly with distance). The clothoid element is used for joining straight and circular segments, or circular segments with different curve radii. Plotting the road curvature as a function of distance, as in Figure 4, results in a diagram with either horizontal lines, representing straight sections (if zero curvature) and circles, or lines with an inclination representing the clothoids.

![Figure 4. Road curvature (1/m) as a function of distance (m) for part of the road.](image)

There are two common demands on a road construction:

1. The tangent vector is continuous.
2. The curvature is continuous.

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4 http://sv.wikipedia.org/wiki/WGS84
The first demand prevents the driver from being instantaneously directed off track. The second one ensures that the vehicle can follow the road by applying a continuous lateral acceleration. If both conditions are fulfilled the road is said to be G2 continuous.

In contrast, a road with a non-continuous curvature implies that the vehicle will gradually be directed off track when passing a discontinuity.

The problem of modelling the road curvature from a set of GPS coordinates can be formulated as a least square minimization problem; find the number and positions of the knots (indicated as red points in Figure 4) that minimizes the distance between the GPS points and the resulting two-dimensional road stretch. However, due to the complicated representation of clothoid curves in Cartesian coordinates it is a very difficult problem, and despite plenty of research carried out no one has been able to find an optimal solution.

Many different optimization schemes were tried and eventually a novel method (Baran, Lehtinen and Popovic, 2010) turned out to be successful in fitting a clothoid spline to GPS data from NVDB. This new method has been implemented in a computer program called Cornucopia5, which has been used in the Known Roads project.

2.3. Elevation

Theoretically, the road elevation could be determined from integration of the inclination measured by the RST vehicle. However, from comparisons of measurements conducted in opposite directions of the same road it was evident that the drift of the inclinometer sensor was too large, leading to a substantial deviation of the elevation compared to reality. This is a general problem for inclination measurements. It was not possible to cancel the drift by removing low frequency content, so another approach was necessary.

Since the surrounding terrain also was going to be modelled, terrain elevation data had already been acquired from Lantmäteriet. Using the continuous description of the road’s horizontal positions from the clothoid spline, together with the flight height data, allowed the road elevation at any point to be determined from bilinear interpolation. This elevation, however, only contains low frequency information so high frequency elevation information from the RST inclinometer measurements had to be superimposed in order to reach the desired level of detail, as described below.

2.4. Road surface

In addition to the elevation, which is a measure that only applies to the centre of the road, there are three other important properties which are measured for each lane individually:

- High frequency elevation for each lane.
- Crossfall for each lane.
- Road irregularities within each lane.

The high frequency elevation is constructed from high pass filtering of the integrated measured inclination. The sampling length is 0.1 m, and a filter cut-off length of 10 m was used for the filtering. For the measured road the maximum amplitude of the high frequency elevation is about 0.1 m.

The crossfall is an angle specifying the lateral inclination of a road lane. It is determined at each sample point from the 17 laser data points using linear curve fitting, as illustrated in Figure 5.

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5 http://code.google.com/p/cornucopia-lib/
After subtracting the crossfall from the laser tracks only the road irregularities are left. This is illustrated in Figure 5, where laser data for a sample taken in a left turn is shown. After removing the crossfall two typical rut tracks with a depth of about 1 cm appear.

2.5. Lane merging

To match the two separate measurements of the lanes is similar to the problem of matching the RST data with the road’s clothoid description. The measurements of two lanes along a road segment usually have different lengths due to the fact that inner curves are shorter than outer curves. Thus, even if the lanes are well matched at one point, alignment will shift along the road depending on the road curvature. The two measurements were made with slight lateral overlap, allowing a comparison of the measurements, which revealed alignment discrepancies. An attempt was made to re-scale the measurement distances by projecting the path of the vehicle on the centre line of the road for both measurements. This resulted in a better match between the measurements, but it was far from good. Instead, a synchronization algorithm was developed where identical peaks are identified within low-pass filtered versions of the measurements, allowing for a variable scaling of the sample point positions for one of the measurement files in order to match the other. The method works well, and while being less sophisticated than e.g. Dynamic Time Warping, it is not as computationally demanding.

After merging the two RST measurements into one data set, it had to be matched with the derived clothoid description. GPS matching from the two sources is not accurate enough and again the synchronization algorithm was used. A very good agreement was found between the curvature measurements from the RST and the clothoid description, making a match of the data sets feasible. A good agreement between RST data and road curvature is necessary since the road crossfall is strongly connected to road curvature.

2.6. Road representation

To be able to use the road description in the driving simulator, the data had to be represented in the form of OpenDrive\textsuperscript{6} and OpenCRG\textsuperscript{7} models. These are two international open file format standards for

\textsuperscript{6} \url{http://www.opendrive.org/}

\textsuperscript{7} \url{http://www.opencrg.org/}
road representation in driving simulators, where the former describes the main properties of the road and the latter the high frequency road surface information.

In OpenDrive road curvature, elevation and crossfall are described analytically as spline functions. While the road curvature already was described analytically, the elevation and crossfall data was not. Spline fitting can be cumbersome, especially with boundary conditions and demands of continuous derivative (denoted C1 continuity). Initially, this was too difficult and instead spline fitting using cubic polynomials with C0 continuity was used. However, trials in the driving simulator using Volvo’s vehicle model of an A-double truck trailer combination showed that C1 continuity was indeed necessary as heavy bumps otherwise would be present. Most likely these bumps can be attributed to a too simplified vehicle model using rigid wheels, but this has not been further investigated.

A method for spline fitting of cubic C1 polynomials with boundary conditions was therefore developed in-house and used for representing elevation and crossfall within the OpenDrive format. In addition, the finer structure of the road surface from the laser measurements was also fitted using the same method. By specifying a desired tolerance of how well the fitting represents the actual data points, it is possible to model the measured data at the desired accuracy. For road 180, which has been used as an example in this report, a tolerance was set that resulted in a discrepancy consisting of noise with a maximum amplitude of 5 mm. This discrepancy was stored as a large data grid within the OpenCRG format.

2.7. Validation with respect to road induced vibrations

Tests were conducted in VTI’s latest driving simulator, Sim IV⁸ (see Figure 6), which is located in Gothenburg, Sweden. It is a moving base driving simulator with interchangeable cabins and has been in operation since May 2011. The motion system combines the possibilities of a hexapod motion base with the extended motion envelope in x and cy directions through a 5x5 m sled system.

![Figure 6. The Sim IV moving base driving simulator at VTI.](http://www.vti.se/en/research-areas/vehicle-technology/vtis-driving-simulators/)

Vibrations from the road will be generated by the hexapod movements, and the performance is shown in Table 1 and Table 2. As evident from the specifications, the acceleration and excursion capabilities of the hexapod should be more than enough to generate road induced vibrations with correct amplitudes. Due to motion cueing algorithms and coupling to the Eigen frequencies of the construction, actual movements may deviate from the vehicle movements in the simulation for certain frequencies. The exact behaviour is still under investigation.

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### Table 1. Hexapod performance.

<table>
<thead>
<tr>
<th></th>
<th>Excursions</th>
<th>Velocity</th>
<th>Accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>-408 / +307 mm</td>
<td>+/- 0.80 m/s</td>
<td>+/- 6.5 m/s²</td>
</tr>
<tr>
<td>Sway</td>
<td>-318 / +318 mm</td>
<td>+/- 0.80 m/s</td>
<td>+/- 6.0 m/s²</td>
</tr>
<tr>
<td>Heave</td>
<td>-261 / +240 mm</td>
<td>+/- 0.60 m/s</td>
<td>+/- 6.0 m/s²</td>
</tr>
<tr>
<td>Roll</td>
<td>-16.5 / +16.5 deg</td>
<td>+/- 40 deg/s</td>
<td>+/- 300 deg/s²</td>
</tr>
<tr>
<td>Pitch</td>
<td>-15.5 / +16.0 deg</td>
<td>+/- 40 deg/s</td>
<td>+/- 300 deg/s²</td>
</tr>
<tr>
<td>Yaw</td>
<td>-20.5 / +20.5 deg</td>
<td>+/- 50 deg/s</td>
<td>+/- 300 deg/s²</td>
</tr>
</tbody>
</table>

### Table 2. Sled system performance.

<table>
<thead>
<tr>
<th></th>
<th>Excursions</th>
<th>Velocity</th>
<th>Accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>+/- 2.5 m</td>
<td>+/- 2 m/s</td>
<td>+/- 5 m/s²</td>
</tr>
<tr>
<td>Sway</td>
<td>+/- 2.3 m</td>
<td>+/- 3 m/s</td>
<td>+/- 5 m/s²</td>
</tr>
</tbody>
</table>

Vibration measurements were conducted in the driving simulator using an iPhone with an app called “Sensor Data”. This allowed accelerations to be measured at 100 Hz in all three directions using the built-in accelerometer of the smartphone. The phone was attached on the panel next to the driver’s chair as shown in Figure 7. Using a smartphone for the measurement was chosen due to its simplicity, facilitating an identical measurement setup for subsequent experiments on real trucks.

![Figure 7. Position of the smartphone in the simulator cabin for measuring vibrations.](image)

Three separate 6 minute long drives along the same road section were carried out at a driving speed of about 60 km/h. Unfortunately, after finishing the measurements it was found that the smartphone had stopped logging data after 60 seconds for each drive, resulting in a large loss of data. Still, the remaining data could be analysed and allowed for some interesting observations.

The measured vertical acceleration of one drive is shown in Figure 8. The plot is representative for all the drives and shows that the maximum acceleration is around 1.5 m/s². In addition to the measured acceleration, the logged simulated vertical acceleration of the truck (taken at a point 1.15 meters above the centre of the front axle) from the driving simulator kernel is shown. There is a very good...
agreement between the simulated vertical acceleration and what was actually measured inside the truck cabin.

**Figure 8.** Vertical acceleration (m/s\(^2\)) plotted as a function of time (s). Measured acceleration (blue) and simulated acceleration (red). Close-up in the right corner.

The energy content at different frequencies is analysed with power spectral density (PSD) calculations. Figure 9 shows the PSD energy content for vibrations in all three directions, with different colours for the three separate measurements.

**Figure 9.** PSD energy content plotted in log scale as a function of frequency (Hz) for vertical (Az), longitudinal (Ax) and lateral (Ay) accelerations (m/s\(^2\)) from three separate measurements (red, blue, green).

Vertical vibrations have a large peak at around 1 Hz, which is to be expected and corresponds to a rigid body vibrational mode. The energy content in this vibration mode is in line with previous experiences on a road with similar roughness as Road 180. There is also energy content, although on a much smaller scale, around 7-8 Hz and 12 Hz. Peaks at those frequencies also show up in the longitudinal and lateral vibrations. It is unclear how to interpret these vibrations. Normally frame structure vibrations show up from 7 Hz and upwards, while front axle vibrations are around 10 Hz. The vehicle model has not been constructed for frame structure vibrations. Front axle vibrations, on the other hand, will likely show up in the simulation. As mentioned above, coupling to the Eigen frequencies of the simulator construction may effectively act as a filter on the produced vibrations, which adds to the complexity of generating the correct forces.

From a vibrational point of view, the vehicle model was primarily constructed for low frequency vibrations in the range from 1 to 3 Hz. In that respect, the road model seems to generate satisfactory results in the driving simulator. Still, for a deeper understanding and improvement of the models, future work should involve real truck measurements on the same road for comparison with the simulator data.
3. Horizontal properties

The horizontal properties of a road can be described by its curvature, as explained in Chapter 2.2. From a set of GPS coordinates describing the road, a horizontal road model can be created using the Cornucopia algorithm (see Chapter 2.2). In Chapter 3.1 to 3.3 the different GPS source data are described and compared. The Cornucopia settings that were used and the outcome of the algorithm applied on Road 180 are also presented in Chapter 3.3.2.

3.1. NVDB

The National Road Database\(^9\) (NVDB) contains a 2-dimensional description of the Swedish road network. Each road is defined by a number of points in SWEREF 99 tm coordinate system\(^10\). The points lay unevenly spread along the road centre chord line, and long straights can be described with few points while curvy parts naturally need more points. This way of representing a road opens up for a natural conversion to clothoid splines.

From our point of view NVDB is the most correct data source for the road’s world position. Deviation from Google Maps (Figure 10) is explained by skewing errors in Google map projections. Even Google’s own road alignment lays off-road compared to satellite photos in Google Maps.

![Figure 10. Road alignment according to NVDB shown on a satellite photo in Google Maps.](image)

3.2. Google

Google can also be used in order to find a road’s alignment in world, by making a travel direction request between the ends of the road, either in Google Maps or Google Earth. The direction can be

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\(^9\) Nationella väg databasen: http://www.nvdb.se/

\(^10\) http://sv.wikipedia.org/wiki/SWEREF_99(TM)
exported to a kml-file (in xml format) in order to be parsed in any software. The coordinates are GPS points in WGS84 system\footnote{http://sv.wikipedia.org/wiki/WGS84}.

The data set of Google is like NVDB non-uniformly sampled, i.e. relatively few data points constitute the road path accurate enough. However, we have seen examples where Google's direction deviates from true alignment (according to NVDB).

3.3. Methodology

3.3.1. Ocular investigation and adjustment of data points

We assume that NVDB data describes the correct road centre line. However, using Google Earth it can be seen that even NVDB data may have different discrepancies. In Figure 11 we can see that the start of the path deviates from the satellite photo path. This can possibly be explained by our NVDB data being outdated, and that the road at some stage has been re-built.

![Figure 11. NVDB data points for a road segment in Borås shown on a satellite photo in Google Earth.](image)

The points that with certainty can be concluded to deviate from proper alignment must be manually rejected or adjusted, either visually with the help of satellite photos or supported by other measurements (e.g. RST or Google directions).

This step is important because it actually synchronises the road model’s geographical position with the real world around us. Since other data sources are used, such as Lantmäteriets elevation database or buildings map (Fastighetskartan), there is otherwise a risk that the road is ported into the surrounding terrain or cuts through a wall corner.
3.3.2. Cornucopia

The following section is based on the research paper "Sketching Clothoid Splines Using Shortest Paths" (Baran et al., 2010). The paper describes an algorithm (the Cornucopia algorithm) that converts handwritten curves into a spline of clothoids, a problem similar to ours. In their case a number of sampled screen pixels has to be converted into analytical splines, while in our case it is sampled geographical points that have to be converted. Source code for Cornucopia is available to download via a so-called Mercury repository. To adapt the Cornucopia to large-scale wave modelling (compared to small sketching on a computer screen) some algorithm parameters had to be adjusted, see settings in Table 3.

Table 3. Parameter settings for the Cornucopia algorithm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic set</td>
<td>Pre-set</td>
<td>ACCURATE</td>
</tr>
<tr>
<td>PIXEL_SIZE</td>
<td>Parameter</td>
<td>0.4</td>
</tr>
<tr>
<td>MAX_RESCALE</td>
<td>Parameter</td>
<td>1.0</td>
</tr>
<tr>
<td>LINE_COST</td>
<td>Parameter</td>
<td>7.5</td>
</tr>
<tr>
<td>ARC_COST</td>
<td>Parameter</td>
<td>9.0</td>
</tr>
<tr>
<td>CLOTHOID_COST</td>
<td>Parameter</td>
<td>15.0</td>
</tr>
<tr>
<td>G0_COST</td>
<td>Parameter</td>
<td>Infinity</td>
</tr>
<tr>
<td>G1_COST</td>
<td>Parameter</td>
<td>Infinity</td>
</tr>
<tr>
<td>G2_COST</td>
<td>Parameter</td>
<td>0</td>
</tr>
</tbody>
</table>

By using a lower value on PIXEL_SIZE the algorithm sensitivity was increased. MAX_RESCALE was changed to 1.0, which produced better results (without a deeper knowledge why). Using the ACCURATE pre-set resulted in an accurate fit.

The settings (in Table 3) for G0_COST, G1_COST and G2_COST result in a G2-continuous road description, which is what we demand. The outcome of the Cornucopia program is a spline of clothoids (including straight segments and curves with constant radius) which describes the road’s horizontal properties in a parametrical analytical way.

Analysing the Cornucopia outcome for the 37 km long Road 180 between Borås and Alingsås shows that 300 line segments in total are demanded, of which 25 are straight segments, 25 curves with constant curve radius, and the rest (greater part) clothoids, see Figure 12. The average segment length is 117 m, and the range is from 22 m to 1083 m, see Figure 13. The average discrepancy from the NVDB data points is 0.2 m, and the maximum error is about a meter with the settings in Table 3 above. The data fitting can be done within 60 seconds on a quite powerful desktop computer.

A disadvantage of the Cornucopia algorithm is that it cannot take boundary values into account. Since a road network normally consists of a number of shorter roads connected in various ways, Cornucopia results of different roads must somehow be stitched together. This problem can be overcome by making separate Cornucopia fittings that overlap each other by a few hundred metres, and then cut the solutions and join them by using a connecting segment. This reduces the problem to that of finding a connecting segment that fulfils the boundary values (G2 continuity) of the two roads that are being

---

12 http://code.google.com/p/cornucopia-lib/
joined. However, finding such a connecting segment is a non-trivial problem which is discussed in detail, together with a possible solution, in the Appendix.

Figure 12. Distribution of different segment types for the 37 km long Road 180 between Borås and Alingsås.

Figure 13. Distribution of segment lengths for the 37 km long Road 180 between Borås and Alingsås.
4. Elevation

4.1. Road elevation from terrain elevation data

Terrain elevation data has been bought from Lantmäteriet. We have height data grids with a grid sample length of 2 meters. We have no information how the data has been pre-processed. Possibly some filtering may have been applied. Each grid is 1250 x 1250 meters in size.

4.2. Elevation example: Road 180

The road is 37 km long and runs along several height data grids as shown in Figure 14.

![Figure 14. The data grids with height information (m, both axes). Road 180 is shown inside the grid.](image)

The elevation of Road 180 was generated from interpolating the elevation values of the grids using a road representation where the Cornucopia fit of the NVDB GPS coordinates has been sampled with a sampling length of 0.1 m. Bilinear and bicubic interpolations were compared and it was concluded that the former method is preferable. The preference for bilinear interpolation can be explained by the fact that noise and irregularities from the grid elevation are introduced in the interpolated elevation, and needs to be removed. The bilinear interpolation is less sensitive to these irregularities than the bicubic method.

The origin of the irregularities is most probably a broad filtering of the grid elevation data before it was supplied to us from Lantmäteriet.

The following pages include four separate examples that illustrate common types of irregularities together with photos of the real road. Since the irregularities cannot be removed by normal low-pass filtering, a manual procedure was necessary. To aid in that process, the derivative of the elevation (i.e. the inclination) was plotted on top of the elevation curve. Figure 15 to Figure 17 show how the elevation from the bilinear and bicubic interpolations look like in general at different length scales. An overview of the elevation is seen in Figure 15. Close-ups of this figure (in Figure 16 and Figure 17) reveal that there are three overlapping curves (only one seen in Figure 15 due included distance).
Figure 15. Elevation (m) of Road 180, only one curve seen due to long distance on x-axis (36 km).

As seen in the close-ups, in Figure 16 and Figure 17, there are a lot of ripples present in the interpolation data, regardless whatever interpolation method that is used. The inclination, indicated by the black line, is normalized with respect to the elevation (using the bilinear interpolation, blue curve). The vertical distance between the black and blue curves gives the inclination, where 0.1 meter difference corresponds to 1% inclination.

Figure 16. Elevation (m) of Road 180 - close-up (distance on x-axis 400 m).
4.2.1. Example 1 – A dip of 0.5 m

A dip of 0.5 m can be seen in Figure 18. The inclination is very steep, about 20%, which makes this dip suspicious. By inspection of photos of the road (in Figure 19), it was concluded that the dip is spurious and needs to be removed.

Figure 18. Elevation dip of 0.5 m (red and blue).
Figure 19. Photos of the road section that corresponds to the position of the dip in Figure 18.
4.2.2. Example 2 – A very steep dip of 0.5 m

A steeper dip with an inclination of about 50% can be seen in Figure 20. The dip is about 4 m long and 0.5 m deep. Photo inspection of the road (see Figure 21) shows no sign of an existing dip. However, the road passes over a bicycle tunnel and it is plausible that pre-filtering of the grid elevation data has smeared elevation from the terrain surrounding the road onto the actual road.

![Figure 20](image)

**Figure 20.** Elevation of a very steep dip of 0.5 m (red and blue).

![Figure 21](image)

**Figure 21.** Photo of the road section that corresponds to the position of the dip in Figure 20.
4.2.3.  Example 3 – Passing over a narrow stream of water

In the next example the road passes over a narrow stream of water, which is indicated by the temporarily increased height of the guard rails in the photo in Figure 23. The impact on the road elevation is the steep dip, about 10 m long and 2 m deep, shown in Figure 22, which needs to be removed. It is clear from Figure 22 that the bicubic interpolation results in unnecessary ripple at the edge of the dip, and that bilinear interpolation is preferable.

Figure 22. A steep dip of 2 m (red and blue).

Figure 23. Photo of the road section that corresponds to the position of the steep dip in Figure 22.
4.2.4. Example 4 – Ripples due to mismatch

Next example is an illustration of ripples that can be introduced in the interpolation (Figure 24). In this case the irregularities are likely due to large stones close to the roadside, see Figure 26. This example also illustrates another effect, namely that the NVDB coordinate system and the elevation data coordinate system may sometimes be mismatched. The red line in Figure 25 and Figure 26 is the NVDB road centre line plotted in the Google Maps photo. Although the apparent mismatch is between NVDB and Google Maps coordinates, and not Lantmäteriets elevation data, there is no guarantee that mismatch between NVDB and the latter does not exist.

In order to minimize mismatch, a tool that can display a 3D map of the elevation together with the NVDB coordinates would be very useful. This was however not possible to develop within the resources of the Known Roads project.

Figure 24. Road elevation – ripples.

Figure 25. Photo of the road section that corresponds to the graph in Figure 24.
After irregularities had been manually removed, the elevation profile was low-pass filtered using a phase-less Butterworth low-pass filter, with a 50 m cut-off length. The results are illustrated in Figures 27 to 29 below. It is important to inspect the resulting inclination of the elevation profile to make sure that all irregularities have been removed.

The generated elevation profile in Figure 28 is a smooth road that fits the surrounding terrain elevation as described by Lantmäteriets elevation grid. It should be noted that wavelengths much shorter than 50 m do not exist in the elevation profile. So while it may be used for many simulation studies, including fuel consumption estimation, the road description will not be detailed enough to introduce high frequency vehicle vibrations.

The shorter road elevation wavelengths must be deduced from the RST measurements. The specific RST measurement that refers to the roads centre line elevation is the inclinometer data. Due to sensor drift this data cannot be used to generate the entire road elevation profile, but if the data is high pass filtered (or band pass filtered) the low frequency information can be removed and the remaining short wave elevation can be added to Lantmäteriets elevation profile. The sampling length of the RST measured elevation is 0.1 m, and a filter cut-off length of 10 m was used for the filtering. For the Road 180 example, the maximum amplitude of this high frequency elevation was about 0.1 m.
Figure 27. Example of manual removal of an irregularity (blue).

Figure 28. Elevation profile before (red) and after (black) low-pass filtering.

Figure 29. Resulting inclination (%) after low-pass filtering. Road 180 between Borås and Alingsås (37 km).
The elevation profile is still in discrete form (0.1 m sample length) and a polynomial spline representation must be created in order to describe the elevation in the OpenDrive format. This process is exemplified in Chapter 4.3 below.

### 4.3. Polynomial approximation

A 3\textsuperscript{rd} degree polynomial spline representation is created using an algorithm developed by VTI. The algorithm can produce splines with continuous first derivative (C1 continuity), and represents a best fit without necessarily passing through the data points. Hence, a spline description with quite a few elements can be produced. The accuracy of the fit can be set by adjusting a tolerance setting, and the difference between the estimated height data and the polynomial approximation can thus be controlled. In general, a better fit requires more spline elements. It is desirable to have a representation with rather few elements in order to have a faster realization in the driving simulation. The fitting is done on the inclination profile (the derivative of the elevation), which ensures that the inclination is proper. The results are exemplified for Road180.

With 1029 spline elements (about one element each 40 meters) the polynomial elevation is within 0.1 m of the terrain data. A comparison between estimated elevation and polynomial fit (approximation) for a part of the road is shown in Figure 30 together with the elevation difference for the entire road.

**Figure 30. Elevation polynomial spline fit of Road 180 using 1029 spline elements.**

With 1550 spline elements the fitting is better (see Figure 31), and the maximum difference between estimated elevation and polynomial approximation is 0.034 m.

**Figure 31. Elevation polynomial spline fit of Road 180 using 1550 spline elements.**
A drawback with the algorithm used in the examples above is that it is difficult to match boundary values. In order to connect separate roads in a network both road elevation and inclination must be continuous at the boundary points. Thus, an alternate algorithm was developed that was able to take these boundary conditions into account.

4.4. **Bridges and tunnels**

When modelling bridges and tunnels, the elevation cannot be constructed from Lantmäteriets elevation data. Instead, some standard bridge and tunnel elements must be used, and the elevation profile of these elements must be connected to roads in the road network. These bridge and tunnel elements can easily be constructed using just a few polynomials. Since the boundary conditions are C1 only it is straightforward to analytically derive the polynomials using for example equation 19 in the Appendix.
5. Road surface

The construction of the road surface, which includes crossfall and road irregularities are described in Chapters 2.4 and 2.5. The derived road descriptions are then implemented according to standard OpenDrive and OpenCRG models, as described in Chapter 2.6.
References


Appendix – Mathematical Modelling

General definitions

A curve can be described in parametrical form as a function of arc length $s$ by its position vector $\vec{r}(s)$

The curve tangent at a specific point is:

$$\vec{T}(s) = \frac{d\vec{r}}{ds} \quad (1)$$

The curvature $\kappa$ is defined by

$$\frac{d\vec{T}}{ds} = \kappa \hat{n} \quad (2)$$

where $\hat{n}$ is a vector orthogonal to $\vec{T}$.

The notion of curvature measures how sharply a curve bends. We would expect the curvature to be 0 for a straight line, to be very small for curves which bend very little and to be large for curves which bend sharply. If we move along a curve, we see that the direction of the tangent vector will not change as long as the curve is straight. Its direction will change if the curve bends. The more the curve bends, the more the direction of the tangent vector will change, motivating the definition above.

It can be shown that the curvature of a circle with radius $a$ is $\kappa = 1/a$. Hence it makes sense to define the curve radius of any point of the curve as the inverse of the curvature.

The definitions above are made with the curve expressed as a function of arc length. Since the arc length can be expressed as a function of any parameter $t$, the curvature can be calculated also for the more general curve definition $\vec{r}(t)$. It can be shown that:

$$\kappa = \left\| \frac{\vec{T}(t)}{\| \vec{T}(t) \|} \right\| \quad (3)$$

Where the dot notation for derivative has been used, and the double bars indicate vector norm. Using that the tangent vector $\vec{T} = \ddot{\vec{r}}/\| \ddot{\vec{r}} \|$, this can be rewritten as

$$\kappa = \left\| \frac{\ddot{\vec{r}}(t) \times \dddot{\vec{r}}(t)}{\| \dddot{\vec{r}}(t) \|} \right\| \quad (4)$$

Introducing a vector $\vec{b}$, defined by $\hat{n} = \vec{b} \times \vec{T}$, equation 4 can be written as

$$\kappa \vec{b} = \frac{\ddot{\vec{r}} \times \dddot{\vec{r}}}{\| \dddot{\vec{r}} \|} \quad (5)$$
Some useful alternate formulations

\[
\kappa \hat{n} = \frac{\ddot{r} - \ddot{T} (\ddot{T} \cdot \hat{T})}{\ddot{T} \cdot \dot{T}} \quad (6)
\]

\[
\kappa = \frac{\ddot{T} \cdot \hat{n}}{\ddot{T} \cdot \dot{T}} \quad (7)
\]

Of special interest for road generation is the formulation in the 2-dimensional xy-plane, where \( \vec{r}(t) = (x(t), y(t)) \)

\[
\kappa(t) = \frac{\ddot{x}y - \ddot{y}x}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \quad (8)
\]

The sign of the curvature:
In OpenDrive curvature of a left curve is defined as positive. This means that the curvature should be positive if \( \vec{b} \) is in positive \( \hat{z} \)-direction (with xyz being an orthonormal right handed system with xy in the road plane).

Clothoids
A clothoid is a curve which curvature is changing linearly with arc length. Representing a clothoid in Cartesian xy-coordinates are done using Fresnel integrals, which must be evaluated numerically.
How to make transitions between different road segments

It is clear that there is a need to connect road segments that have been constructed separately. For a perfect connection, we demand G2 continuity, which means that not only shall the tangent vectors of the connected parts match, but also the curvature needs to match.

One way to do this would be to use clothoid splines for this connection. Another method that is supported by OpenDrive, is the use of 3rd degree polynomial splines. The clothoid solution has the advantage that the resulting road changes its curvature linearly with distance, which is the way that most real roads are constructed. If, for some reason the clothoid solution does not result in an attractive solution, it may be possible to use the other method, i.e. a polynomial description for modelling parts of the road network. Below we present algorithms for both types of connections, and discuss their pros and cons.

General problem

Two points, $P_0$ and $P_1$ shall be connected. The tangent vectors for the two points, $k_0$ and $k_1$ are given. The curvatures, $\kappa_0$ and $\kappa_1$ are also known. In the following text vectors are indicated with bold text, as opposed to scalars. We are looking for a curve that joins the two points and which results in continuous tangents and curvature.

Solution 1: Use clothoid splines

The general problem above has been addressed by Meek and Walton [1] using clothoid spline spirals for the transition. A spiral is a curve whose curvature is monotone increasing or decreasing and does not change sign. Note that a spiral cannot reproduce an s-shaped curve, which would be needed if $k_0$ and $k_1$ have different signs. They prove that a clothoid spline spiral solution that obeys the boundary conditions is possible, if and only if certain conditions are fulfilled. Those conditions are quite broad, and does allow for solutions for many cases. Moreover, they also prove that there is only one unique solution to the problem. This unique solution always consists of three spline segments. The splines can be of three different types:

1. Straight line -> clothoid -> circular arc
2. Circular arc -> clothoid -> circular arc
3. Clothoid -> circular arc -> clothoid

where the type 1 of course can exist in reverse form. In the paper algorithms are presented for how to determine which type of solution that is applicable to specific boundary conditions, together with algorithms for how do determine the splines. The solutions cannot be determined analytically, and thus numerical methods needs to be applied.
We found that the most straightforward method was to construct splines of each type, fulfilling the curvature boundary conditions and starting in $P_0$ with direction $k_0$, and then for each type perform a multivariate optimization and minimize the discrepancy for the resulting $P_1$ and $k_1$. Reasonable start values for the spline segments lengths could be found from constructing the quadratic Bezier curve connecting the points and then split its length into three equal parts. Higher order Bezier curves may be needed in some cases, and although the Bezier curve is simpler than the clothoid, it is not straightforward to fulfill G2 continuity with the Bezier description. See the section about Bezier curves at the end of this Appendix.

For the case where $k_0$ and $k_1$ have different signs, an s-shaped solution is needed. We have found that a general clothoid spline of length 3 will result in optimized solutions for the majority of transitions we were interested in, including s-shaped conditions. For the cases that did not result in solutions, extending the clothoid spline to length 4 or 5 solved the problem.

Solution 2: Use polynomials

In OpenDrive a roads geometry in the horizontal plane may be described by a series of 3rd degree polynomials, commonly referred to as cubic splines (or just csplines). There exist powerful methods in the literature on how to join several points together, and it is possible to demand G1 and G2 continuity.

![Spline interpolation (from Wikipedia).](image)

However, these methods describe the spline in parametric form (x and y as functions of a parameter, often denoted t). Unfortunately, this is not supported by OpenDrive. In OpenDrive, each spline is defined in a local Cartesian coordinate system $uv$, aligned so the starting tangent vector is pointing in the $u$-direction. It is clear that a general spline in parametric form cannot be transferred to a function in the $uv$-system. Thus, another solution must be found. Below we describe a method that we have developed.
Consider first a cubic polynomial in a Cartesian $uv$-system with origin in $(0,0)$, and its starting tangent vector $k_0$ in the $u$-direction, according to Figure 34. The curvature at $(0,0)$ is fixed and denoted by $\kappa_0$.

Let $P = (u_p, v_p)$ be a point on the curve. First, it is clear that $P$ has to be positioned to the right of the $v$-axis in order to find a function passing through it.

The general form of a cubic polynomial is:

$$v = a_0 + a_1 u + a_2 u^2 + a_3 u^3$$  \hspace{1cm} (9)

The first and second derivative:

$$v' = a_1 + 2a_2 u + 3a_3 u^2$$ \hspace{1cm} (10)

$$v'' = 2a_2 + 6a_3 u$$ \hspace{1cm} (11)

The curvature of a general function is given by

$$\kappa = \frac{v''}{(1 + v'^2)^{3/2}}$$ \hspace{1cm} (12)

The boundary values $v(0)=0; v'(0)=0$ make $a_0 = a_1 = 0$

$$\kappa_0 = \frac{v''(0)}{(1 + v'(0)^2)^{3/2}} = v''(0) = 2a_2 \Rightarrow a_2 = \frac{\kappa_0}{2}$$

Thus

$$v = \frac{\kappa_0}{2} u^2 + a_3 u^3$$ \hspace{1cm} (13)

$$v' = \kappa_0 u + 3a_3 u^2$$ \hspace{1cm} (14)

The curvature at an arbitrary point on the curve is given by

$$\kappa = \frac{\kappa_0 + 6a_3 u}{(1 + (\kappa_0 u + 3a_3 u^2)^2)^{3/2}}$$ \hspace{1cm} (15)
Demanding that the curve intersects a specific point \( P = (u_p, v_p) \) uniquely defines the \( a_3 \) parameter according to eq. 5, and we get

\[
a_3 = \frac{v_p - \frac{\kappa_0}{2} u_p^2}{u_p^3}
\]

Hence, the tangent vector and curvature in point \( P \) are also uniquely defined and it is therefore in general impossible to connect two points with a single cubic polynomial if one wants to assign specific values to tangent and curvature in the second point.

We show below that in general we need a polynomial spline of minimum length 2 to fulfil the boundary conditions, and often longer splines than that may be required if G1 and G2 continuity is imposed.

But first look at the special case, where G2 continuity is not required, i.e. no requirements on curvature at start and end point:

Equation 9 and 10 can be written in matrix form as

\[
\begin{pmatrix}
  u_p^2 & u_p^3 \\
  2u_p & 3u_p^2
\end{pmatrix}
\begin{pmatrix}
  a_2 \\
  a_3
\end{pmatrix}
=
\begin{pmatrix}
  v_p \\
  k_p
\end{pmatrix}
\]

(17)

Where \( k_p \) is the inclination in point \( P \). Solving this equation gives

\[
\begin{pmatrix}
  a_2 \\
  a_3
\end{pmatrix}
=
\frac{1}{u_p^3}
\begin{pmatrix}
  3u_p & -u_p^2 \\
  -2 & u_p
\end{pmatrix}
\begin{pmatrix}
  v_p \\
  k_p
\end{pmatrix}
\]

(18)

The curvature at the start point is \( \kappa_0 = 2a_2 \), and the curvature at any other point of the curve can be calculated using equation 15.

This formula can be useful in some cases, although G2 continuity is always preferable. It can readily be extended to include cases where the segments start point \( v_0 \) and inclination \( k_0 \) are not set to zero. Then

\[
\begin{pmatrix}
  a_2 \\
  a_3
\end{pmatrix}
=
\frac{1}{u_p^3}
\begin{pmatrix}
  3u_p & -u_p^2 \\
  -2 & u_p
\end{pmatrix}
\begin{pmatrix}
  v_p - v_0 - k_0 u_p \\
  k_p - k_0
\end{pmatrix}
\]

(19)

with \( a_0 = v_0 \) and \( a_1 = k_0 \).
Consider a polynomial spline consisting of 2 polynomials, as in Figure 35. Here, an intermediate point has been introduced. Each polynomial has 4 degrees of freedom due to the unknown parameters \( a_0 \) to \( a_3 \), which in total results in 8 unknown variables. The first polynomial has to match 4 boundary conditions: the curve has to pass through \( P_0 \) and \( P_{\text{int}} \), with a specific start tangent vector and curvature. The second polynomial has to pass through \( P_{\text{int}} \) and \( P_1 \), and must match the end tangent vector and curvature of the first one, as well as the tangent vector and curvature of point \( P_1 \), which results in another 6 boundary conditions.

So in total there are 8 unknown variables and 10 boundary conditions. Regardless of how many splines that are used there will always be 2 more boundary conditions than unknown variables. The problem can be solved with a unique solution if the position of the intermediate point is treated as unknown, which introduces 2 more degrees of freedom as the point is free to move in both \( x \)- and \( y \)-direction. However, since there are restrictions for how much this intermediate point can move for allowing a polynomial description in the local coordinate systems, it may still not be possible to find a solution in the general case.
Bézier curves

Bézier curves are often used in computer graphics and path generation, and may be a useful tool for generating transition curves. They are not supported by OpenDrive, but may since the curvature at any point of a Bézier curve is readily calculated, they may serve as a good starting point for generating a clothoid spline. In this chapter the fundamentals of Bézier curves are presented, which is needed for constructing Bézier splines that solves the general problem of joining two curves, illustrated in Figure 32.

A Bézier curve is described by a number of control points, where the curve passes through the first and the last control point. A curve is said to be of degree \( n-1 \), where \( n \) is the number of control points. For reasons explained below, a Bézier curve of degree 2 is called quadratic, degree 3 is called cubic, and so on. The equation for the Bézier curve resembles that of the equation for the centre of mass for a set of point masses. Consider the points shown in Figure 37 (a).

![Bézier curves in terms of centre of mass](image)

The centre of mass \( \bar{P} \) is

\[
\bar{P} = \frac{\sum_{i=0}^{3} m_i \bar{P}_i}{\sum_{i=0}^{3} m_i}
\]

Now, let the masses be a function of a parameter \( t \), so that

\[
m_0(t) = (1-t)^3, \quad m_1(t) = 3t(1-t)^2, \quad m_2(t) = 3t^2(1-t), \quad m_3(t) = t^3
\]

The sum of the masses always equals one, regardless of the value of \( t \). As \( t \) varies between 0 and 1, the centre of mass will move and sweep out a curve shown in Figure 37 (b).
Different weight (or blending) functions can be used, although the ones described above, denoted Bernstein polynomials, are the most common ones. Bernstein polynomials of general degree $n$ can be constructed by evaluating

$$((1-t)+t)^n = (1-t)^n + \binom{n}{1}(1-t)^{n-1}t + ... + t^n,$$

where each term in the sum is a polynomial of degree $n$, and the sum of polynomials always equals 1. Examples of Bézier curves of different degrees are shown in Figure 38.

![Bézier curves of different degrees](image)

**Figure 38. Bézier curves of different degrees. (from ref [2]).**

Bézier curves constructed using Bernstein polynomials have some interesting properties:

1. The curve always starts at point $P_0$ and ends at point $P_n$.
2. The tangent vector at the starting point will be pointing in the direction of $P_1-P_0$.
3. The tangent vector at the end point will be pointing in the direction of $P_{n-1}-P_n$.

It is clear that a Bézier curve of degree $n$ is a sum of polynomials in $t$ of degree $n$. It can be written as:

$$\mathbf{B}(t) = \sum_{i=0}^{n} b_{i,n}(t)\mathbf{P}_i, \quad t \in [0,1] \quad (19)$$

$$b_{i,n}(t) = \binom{n}{i}(1-t)^{n-i}t^i \quad (20)$$

with $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ being the normal binomial coefficient, and $\mathbf{P}_i$ are the control points.
The derivative with respect to $t$ is

$$\dot{\vec{B}}(t) = \sum_{i=0}^{n-1} n(\vec{P}_{i+1} - \vec{P}_i) \cdot b_{i,n-1}(t) \quad (21)$$

The second derivative is

$$\ddot{\vec{B}}(t) = \sum_{i=0}^{n-2} n(n-1)(\vec{P}_{i+2} - 2\vec{P}_{i+1} + \vec{P}_i) b_{i,n-2}(t) \quad (22)$$

In particular:

\[
\begin{align*}
\dot{\vec{B}}(0) &= n(\vec{P}_1 - \vec{P}_0) \\
\dot{\vec{B}}(1) &= n(\vec{P}_n - \vec{P}_{n-1}) \\
\ddot{\vec{B}}(0) &= n(n-1)(\vec{P}_2 - 2\vec{P}_1 + \vec{P}_0) \\
\ddot{\vec{B}}(1) &= n(n-1)(\vec{P}_n - 2\vec{P}_{n-1} + \vec{P}_{n-2})
\end{align*}
\]

The curvature of a Bezier curve at any point can be calculated using the general expression

$$\kappa \vec{B} = \frac{\vec{r} \times \vec{\ddot{r}}}{|\vec{r}|^3}$$

References:


ViP is a joint initiative for development and application of driving simulator methodology with a focus on the interaction between humans and technology (driver and vehicle and/or traffic environment). ViP aims at unifying the extended but distributed Swedish competence in the field of transport related real-time simulation by building and using a common simulator platform for extended co-operation, competence development and knowledge transfer. Thereby strengthen Swedish competitiveness and support prospective and efficient (costs, lead times) innovation and product development by enabling to explore and assess future vehicle and infrastructure solutions already today.